RELATIONSHIPS FOR A FLUIDIZED BED OF SOLID PARTICLES

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UDC 532.529.5

A new theory for a fluidized bed is presented and corroborated by experimental data.

An ascending stream of fluid brings a bed of solid particles into a fluidized state when the resistance to flow past the particles becomes equal to their weight. With further expansion of the bed the velocity of hindered fall of the particles increases, tending to the free-fall velocity of an individual particle when m → 1. Thus, the boundary states of a fluidized bed are a fixed bed with water filtering through it just prior to the onset of fluidization and free-falling particles. Hence, it is worth while examining some features of these boundary states.

Plots of the resistance coefficients of the particles against the Reynolds numbers for free-falling particles and particles in a fixed bed are of the same nature, as Fig. 1† shows [1, 2]. For equal values of Re_p and Re_b, $\psi_b > \psi_p$, and $Re_b^* > Re_p^*$. Hence, the flow in the bed becomes laminar and the friction force increases. The reason for this is the molar viscosity ν_M [3, 4], due to transfer of momentum between macromasses of the fluid resulting from the frequent and sharp changes in the cross sections and velocities of the local flow components in the bed. This is confirmed by the fact that for the same ordinates on curves 1 and 2 the ratio of the abscissas is ν_0/ν_M,‡ i.e.,

$$\frac{Re_p}{Re_b} = \frac{\nu_0}{\nu_M}. \qquad (1)$$

For particles in a bed the Reynolds number corresponding to the actual conditions of flow past them, i.e., with ν_M taken into account, is

$$Re_M = \frac{V_b d}{\nu_M} = Re_b \frac{\nu_0}{\nu_M}. \qquad (2)$$

According to the above and from a comparison of expressions (1) and (2) it follows that when $Re_M = Re_p$, $\psi_b = \psi_p$, and $Re_M^* = Re_p^*$, i.e., when the Reynolds number for a particle in a bed is determined from formula (2), the relationship $\psi_b = f(Re_M)$ will be the same as $\psi_p = f(Re_p)$. This is clearly illustrated by curve 3, obtained by using expression (2) to convert the abscissas of line 2 (the slight differences between lines 3 and 1 can be attributed to difference in the grain shape and partly to the methods of determining their diameter in the experiments of Zegzhda and Lomize, which are compared in Fig. 1).

The considered relationship can be used to determine ψ_b and Re_b^* from the value of Re_M and the relationship $\psi_p = f(Re_p)$ if the separately falling particles and particles in the bed have the same shape.

† The quantities illustrated on the graph are given by the following expressions:

$$\psi_p = \frac{\pi d g(\rho_p - \rho)}{6 V_p^2 \rho};\ Re_p = \frac{V_p d}{\nu_0};\ \psi_b = \frac{\pi d g m(\rho_p - \rho)}{6 V_b^2 \rho};\ Re_b = \frac{V_b d}{\nu_0}.$$

‡ In Lomize's experiments m = 0.37 on the average, which corresponds to $\nu_0/\nu_M = 0.04$.

All-Union Scientific-Research Institute of Railroad Transport, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 6, pp. 1122-1130, June, 1970. Original article submitted June 2, 1969.

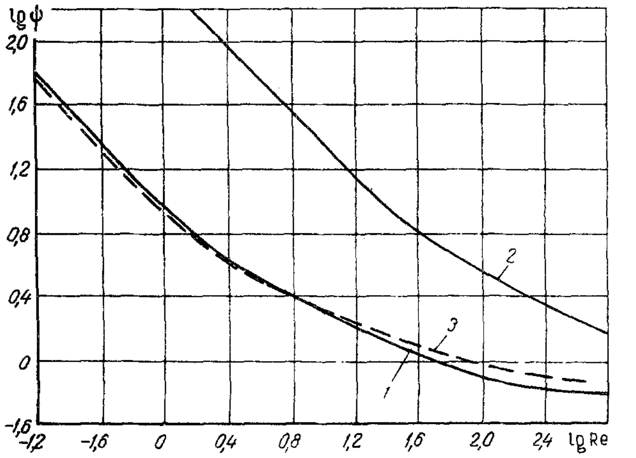

Fig. 1. Resistance coefficients as functions of particle Reynolds numbers: 1) $\psi_p = f(Re_p)$; 2) $\psi_b = f(Re_b)$; 3) $\psi_b = f(Re_M)$.

According to expressions (1) and (2)

$$Re_b^* = Re_M^* \frac{\nu_M}{\nu_0} = Re_p^* \frac{\nu_M}{\nu_0}. \quad (3)$$

For free-falling sand and gravel the lower Re_p^* is 1, and the upper is 400 [1]. When m = 0.4, the ratio $\nu_M/\nu_0 = 20.2$ and, hence, from formula (3) the corresponding Re_b^* are 20.2 and 8080. Hitherto there has been no theoretical method of determining these values, and the experimentally obtained [5-9] values (which confirm the above calculated values) could not be explained.

If the Reynolds number of a particle in the bed is determined from V_0, then

$$Re_0^* = \frac{V_0 d}{\nu_0} = \frac{mV_b d}{\nu_0} = mRe_p^* \frac{\nu_M}{\nu_0}, \quad (4)$$

and the values of Re_0^* are 8.07 and 3230.

The values of Re_b^* and Re_0^* found in this way also relate to the start of fluidization, which is also corroborated by experimental data [10, 11]. For this instant the relationship $\psi_b = f(Re_b)$ corresponds to curve 2 in Fig. 1, and with further expansion of the bed and increase in porosity to m = 1 the corresponding curves lie between curves 1 and 2. Each of these curves corresponds to a particular value of m and the corresponding relationship given by formula (1) still holds, i.e., for all values of m we can determine ψ_b from the relationship $\psi_p = f(Re_p)$, and conversion of the abscissas of these intermediate lines by using formula (2) reduces them to the same line 3, which is independent of m.

The value of ν_M, Re_M, and ψ_b are required for determination of the fluidization velocity (or stability limit of the bed) and the velocity V_b. In addition, V_b depends on the so-called equivalent density of the bed. Since this parameter has been rejected hitherto [12], some explanation should be given.

The weight of unit volume of the fluidized bed is

$$\gamma_b = \gamma m + \gamma_p C_0 = \gamma_p - m(\gamma_p - \gamma) \text{ g/cm}^3, \quad (5)$$

and the difference in hydrostatic pressures over the height is

$$\Delta p = \Delta H \gamma_b \text{ g/cm}^2. \quad (6)$$

Hence, a particle in a fluidized bed will be acted on by an upward force

$$F = \gamma_b w, \quad (7)$$

and the weight of the particle in the bed is

$$G_b = w(\gamma_p - \gamma_b). \quad (8)$$

The bulk density of the fluidized bed is determined with an hydrometer and the hydrostatic heat is determined with a piezometer in exact correspondence with the above formulas.

Division of γ_b by g gives a provisional value of ρ_b – the density of the heterogeneous liquid–solid particle system. Its provisional nature is due to the impossibility of transition to an infinitely small volume in an inhomogeneous medium and to some fluctuations of γ_b in short periods of time due to fluctuations in particle concentration. Hence ρ_b is called the apparent, effective, or equivalent density. Since γ_b for given parameters (C_0, γ_p; γ) of the fluidized bed has the same mean value the use of ρ_b as a theoretical quantity is perfectly sound.

When ρ_b (or γ_b) is taken into account, the weight of a spherical solid particle in a fluidized bed is

$$G_b = \frac{\pi d^3}{6}(\rho_p - \rho_b) g = \frac{\pi d^3}{6} mg(\rho_p - \rho). \quad (9)$$

The resistance to hindered fall of the particle is

$$R_b = \psi_b V_b^2 d^2 \rho. \quad (10)$$

Since these forces are equal the velocity of hindered fall of the particles is

$$V_b = \sqrt{\frac{\pi d g m (\rho_p - \rho)}{6 \psi_b \rho}}. \tag{11}$$

Knowing the limiting value of m at which the fixed bed becomes unstable and fluidization begins, and finding ψ_b by the method given above, we can find from formula (11) the velocity V_{sl} corresponding to this instant. We can also use for the same purpose an expression based on formula (11)

$$Re_{sl} = \frac{V_b d}{\nu_M} = 0.72 \sqrt{\frac{Ar_M}{\psi_b}}, \tag{12}$$

where

$$Ar_M = \frac{g d^3 m (\rho_p - \rho)}{\rho \nu_M^2}. \tag{13}$$

The fundamental difference between these expressions and those proposed earlier [13, 14] is the inclusion of the molar viscosity of the bed and the resistance coefficient of the particles. Since the condition for transition to the fluidization state is the equality of the resistance of the particles to their weight, then any other expression for Re_{sl} not involving the coefficient ψ_b, which depends on Re_M and on the shape of the particles, can be used only in a special case where the conditions of the particular experiment are confined within a narrow range.

For a laminar regime of hindered fall of the particles

$$\psi_b = \frac{3\pi}{Re_M} = \frac{3\pi \nu_M}{V_b d} = \frac{3\pi m \nu_M}{V_0 d}. \tag{14}$$

Substituting expression (14) in (11), we obtain for this regime formulas for the velocities of hindered fall of particles in the fluidized bed:

$$V_b = \frac{d^2 m g (\rho_p - \rho)}{18 \rho \nu_M}, \tag{15}$$

$$V_0 = \frac{d^2 m^2 g (\rho_p - \rho)}{18 \rho \nu_M}. \tag{16}$$

The ratios of V_b and V_0 to the free-fall velocity of the individual particles comprising the fluidized bed for the laminar regime are

$$\frac{V_b}{V_p} = \frac{m \nu_0}{\nu_M}, \tag{17}$$

$$\frac{V_0}{V_p} = \frac{m^2 \nu_0}{\nu_M}. \tag{18}$$

A comparison (Fig. 2) of the results of calculation from formulas (14)-(18) with the experimental data of [12, 15] shows a quite satisfactory agreement.

These formulas can be used to determine V_b or V_0 for any expansion of the bed from the known particle parameters (d, γ_p, V_p) and from m in the case of a laminar regime. When m is determined from V_b the relationship $V_b/V_p = m\nu_0/\nu_M = f(m)$, shown in Fig. 2, should be used.

For a laminar regime we obtain from formula (15)

$$Re_{sl} = \frac{V_b d}{\nu_M} = Re_b \frac{\nu_0}{\nu_M} = \frac{Ar_M}{18} = 0.056 Ar_M. \tag{19}$$

In the case of a turbulent self-similar regime of hindered fall ψ_b becomes constant for particles of a given shape, irrespective of the expansion of the bed, including m = 1, and the moment of onset of fluidization [10, 16]. Hence, the values of ψ_b and ψ_p for this regime are equal and, according to formula (11),

$$\frac{V_b}{V_p} = m^{0.5}, \tag{20}$$

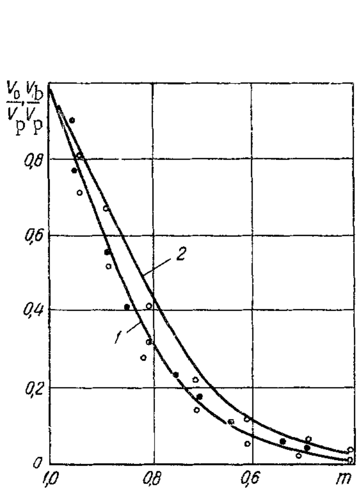

Fig. 2

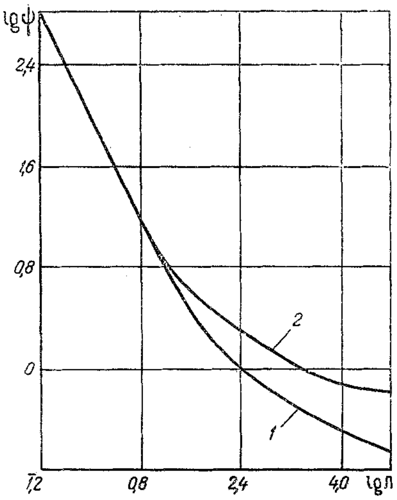

Fig. 3

Fig. 2. 1) Dependence of V_0/V_p on m from formula (18) and from experiments of Richardson and Zaki (black points) and Mints (white points); 2) dependence of $V_b/V_p = m\nu_0/\nu_M$ on porosity of bed.

Fig. 3. Dependence of ψ_p on $Л_p$: 1) for spheres; 2) for sand and gravel.

$$\frac{V_0}{V_p} = m^{1.5}. \tag{21}$$

In a fluidized bed expanding from m_{sl} to m = 1 V_b, Re_b, Re_0, and Re_M vary continuously. As a result, the resistance coefficient ψ_b changes continuously in laminar and transitional regimes even for particles of the same size, density, and shape. The relative simplicity of the relationship between ψ_p and Re_p in the case of laminar flow allowed us to use ν_M and Re_M to obtain analytical expressions of the relationships (14)-(18) for a fluidized bed. In the case of a transitional regime the relationship between ψ_p and Re_p is of a complex nature and is different for particles of different shape. In addition, in fluidized bed conditions we need to find for each new value of m the corresponding values of Re_b and ψ_b. Hence, for calculation of a fluidized bed in the transitional regime the above-discussed method involving the relationship $\psi_p = f(Re_p)$ is the only possible one for particles of a particular shape. It is also necessary to use as auxiliary quantities the Lyashchenko number [17] for a free-falling particle and for a particle in a fluidized bed

$$Л_p = Re_p^2 \psi_p = \frac{V_p^2 d^2}{\nu_0^2} \psi_p = \frac{\pi d^3 g (\rho_p - \rho)}{6\rho\nu_0^2} = 0.52 Ar_p, \tag{22}$$

$$Л_b = Re_M^2 \psi_b = \frac{V_b^2 d^2}{\nu_M^2} \psi_b = \frac{\pi d^3 g m (\rho_p - \rho)}{6\rho\nu_M^2} = 0.52 Ar_M. \tag{23}$$

The ratio of these numbers is

$$\frac{Л_b}{Л_p} = \frac{m\nu_0^2}{\nu_M^2}. \tag{24}$$

Knowing $Л_p$, m, and ν_M, we find the value of $Л_b$, and then from Fig. 3 we determine the corresponding value of ψ_b, which enables us to calculate V_b from formula (11). The graph can easily be constructed if we have the relationship $\psi_p = f(Re_p)$ for particles of the given shape. It should be noted that attempts to use the particle shape factor in such calculations do not help to solve the problem. The value of this factor is determined experimentally and less accurately than ψ_p, and the effect of the shape of the particle on ψ_p depends in a very complicated manner on Re_p. In the case of a fluidized bed there is a further complication due to the dependence on m. The whole procedure is far more difficult and less accurate than direct determination of ψ_p and can only be of illustrative value.

TABLE 1. Regimes of Fluidized Bed of Spheres in Relation to Re_p and m

Re_p	ψ_p	ψ_b	V_b/V_p	Re_M	Re_b	Re_0	Regime of fluidized bed
		for $m=0,43$					
<1	>9,42	>6000	0,026	<0,0016	<0,026	<0,0112	Laminar, m=0.43-1.0
1—110	9,42—0,364	6000—9,42	0,026—0,13	0,0016—1,0	0,026—16,5	0,0112—7,1	Laminar for m=0.43, transitional for $m=m_x$, $m_x=f(Re_p)$
110—1500	0,364—0,16	9,42—0,76	0,13—0,3	1—27,3	16,5—450	7,1—193	Transitional, m=0.43-1.0
1500—38000	0,16	0,76—0,16	0,3—0,65	27,3—1500	450—24750	193—10700	Transitional for m=0.43, turbulent for $m=m_x$, $m_x=f(Re_p)$
>38000	0,16	0,16	0,65	>1500	>24750	>10700	Turbulent, m=0.43-1.0

TABLE 2. Regimes of Fluidized Bed of Sand and Gravel in Relation to Re_p and m

Re_p	ψ_p	ψ_b	V_b/V_p	Re_M	Re_b	Re_0	Regime of fluidized
		for $m=0,4$					
<1	>9,42	>10000	0,02	<0,001	<0,02	<0,008	Laminar, m = 0.4-1.0
1—155	9,42—0,74	10000—9,42	0,02—0,177	0,001—1,0	0,02—20	0,008—8	Laminar for m = 0.4, transitional for $m=m_x$, $m_x=f(Rep)$
155—400	0,74—0,63	9,42—2,76	0,177—0,3	1,0—6,0	20—120	8—48	Transitional, m = 0.4-1.0
400—12700	0,63	2,76—0,63	0,3—0,63	6—400	120—8080	48—3200	Transitional for m = 0.4, turbulent for $m=m_x$, $m_x=f(Re_p)$
>12700	0,63	0,63	0,63	>400	>8080	>3200	Turbulent, m = 0.4-1.0

A simple comparison of the experimentally and theoretically obtained values of Re_b^* and Re_p^* given above indicates that as the bed expands from m_{sl} to m = 1 the regime of hindered fall of the particles can vary. For instance, with $Re_b = 20$ at the start of fluidization of a bed of sand (at m = 0.4) the regime of the bed is laminar, but a further increase in m leads to a transitional regime due to the increase in V_b and Re_b. Hence, it is quite obvious that the regime of a fluidized bed must be determined theoretically by comparing Re_M, found from formula (2), with the lower and upper Re_p^*. It can also be done by finding ψ_b from Re_M and from the graph of the relationship $\psi_p = f(Re_p)$, on which the boundaries of the regimes can be seen. Calculations were made by these methods and Tables 1 and 2, characterizing the different regimes of the fluidized bed in relation to Re_p, Re_b, and m, were compiled.

The data given in the tables shows that maintenance of the same regime at all values of m is a special case of the fluidized state. Such data are of very great importance for determination of the parameters and technological properties of a fluidized bed.

Resistance to the motion of the fluid in a fluidized bed can be determined from the following expression:

$$\gamma\omega L dh = -R ds \tag{25}$$

$$-\frac{dh}{ds} = i = \frac{R}{\gamma\omega L} = \frac{R}{\gamma m L}. \tag{26}$$

We take R equal to the sum of the resistances of the bed particles to the flow of fluid past them, i.e.,

$$R = nR_b = n\psi_b V_b^2 d^2 \rho. \tag{27}$$

Substituting expression (27) and $n = 6C_0/\pi d^3$ in formula (26) we obtain

$$i = \frac{6C_0 \psi_b V_b^2}{\pi d g m}. \tag{28}$$

Replacing V_b by formula (11), we find

$$i = C_0 \frac{\rho_p - \rho}{\rho}. \tag{29}$$

Expressions (28) and (29) are valid for any regime of motion of the fluid in a fluidized bed [18], and the first of them is also applicable to a fixed bed.

Putting i from formula (29) in expression (15) we obtain for the laminar regime

$$V_b = \frac{d^2 m g (\rho_p - \rho)}{18 \nu_M \rho} = \frac{d^2 m g i}{18 C_0 \nu_m}, \tag{30}$$

i.e., a clear linear relationship between the velocity and hydraulic gradient.

Putting i in expression (11) we obtain

$$V_b = \sqrt{\frac{\pi d g m i}{6 C_0 \psi_b}}. \tag{31}$$

This expression, which is common for all regimes with ψ = const, gives a square-law relationship between i and V_b, which is characteristic of a turbulent regime. Formulas (30) and (31) are analogous to the relationships obtained for a flow of fluid in a fixed granular bed [19] on the basis of the considered physical scheme of the effect in relation to ν_M, Re_M, and $\psi_b = f(Re_b)$.

NOTATION

Re_p is the Reynolds number of individual free-falling solid particle;
Re_b is the Reynolds number of particle in fluidized or fixed bed;
ψ_p is the resistance coefficient of individual free-falling particle;
d is the diameter of solid particle;
g is the acceleration of gravity;
ρ_p is the density of solid particle;
ρ is the density of fluid;
V_p is the free-fall velocity of solid particle;
ν_0 is the kinematic viscosity of fluid;
ψ_b is the resistance coefficient of particle in fluidized or fixed bed;
m is the porosity of bed;
V_b is the velocity of flow of fluid in pores of bed (velocity of hindered fall of particle);
Re_p^* is the critical Reynolds number of individual free-falling particle;
Re_b^* is the critical Reynolds number of particle in bed;
ν_M is the molar kinematic viscosity;
Re_M is the Reynolds number of solid particle in bed, determined from molar viscosity ν_M;
Re_M^* is the critical value of Re_M;
Re_0^* is the critical Reynolds number determined from velocity V_0 and viscosity ν_0;
V_0 is the velocity of fluid referred to whole cross-sectional area of bed;
γ_b is the weight of unit volume of fluidized bed;
γ is the weight of unit volume of fluid;
C_0 is the volume concentration of solid particles in bed;
Δp is the difference of hydrostatic pressures;
ΔH is the increment of height in bed;
F is the hydrostatic upthrust acting on solid particle in fluidized bed;
W is the volume of solid particle;

G_b is the weight of particle in bed;
R_b is the resistance to hindered fall of particle;
ρ_b is the equivalent density of heterogeneous system in fluidized bed;
Re_{sl} is the Reynolds number corresponding to stability limit of bed at onset of fluidization, calculated from ν_M and V_b;
Ar_M is the Archimedes number of particle in bed with m and ν_M taken into account;
$Л_p$ is the Lyashchenko number of free-falling particle;
$Л_b$ is the Lyashchenko number of particle in fluidized bed;
Ar_p is the Archimedes number of free-falling particle;
m_{sl} is the porosity corresponding to stability limit of bed;
ω is the cross-sectional area of fluid flow in pores of unit volume of bed;
L is the length of unit volume of bed;
dh is the head in bed required for movement of volume of fluid ωL through distance ds;
R is the resistance to motion of fluid;
i is the hydraulic gradient in bed;
n is the number of solid particles per unit volume of bed;
m_x is the value of porosity m at which hydraulic regime of bed changes.

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